On the form of Lorentz-Stern-Gerlach force

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Abstract

In recent times there has been a renewed interest in the force experienced by a charged-particle with anomalous magnetic moment in the presence of external fields. In this paper we address the basic question of the force experienced by a spin-$\frac{1}{2}$ point-like charged-particle with magnetic and electric moments in the presence of space-and time-dependent external electromagnetic fields, when derived from the Dirac equation *via* the Foldy-Wouthuysen transformation technique. It is interesting to note that the force thus derived differs from the ones obtained by various other prescriptions.

I. INTRODUCTION

We present a derivation of the force experienced by a spin-$\frac{1}{2}$ point-like charged-particle with anomalous magnetic and anomalous electric moments in the presence of space-and time-dependent external electromagnetic fields, based *ab initio* on the Dirac equation *via* the Foldy-Wouthuysen (FW) transformation technique. In the present derivation we neglect the radiation reaction and the electromagnetic fields are treated as classical.

In absence of spin the force experienced by a point-like charged-particle is completely described by the Lorentz force law ($\boldsymbol{F}_L = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$). In the regime where the spin and the magnetic moment are to be taken into account the question of the form of the force obtained from the relativistic quantum theory is still unresolved to this day, though extensive studies, using diverse approaches have been done since the discovery of quantum mechanics. This is evident from the numerous approaches/prescriptions which have been tried to address this basic question and are still being tried. Before proceeding further we note that the expression quoted above constitutes the Lorentz force. The total force which

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we call as the Lorentz-Stern-Gerlach (LSG) force includes the Lorentz force as the basic constituent and all the other contributions coming from the spin, anomalous magnetic and electric moments *etc,*. The reason for this nomenclature will be clear as we proceed.

Here we quote a few approaches which have been used to address the question of the force and acceleration experienced by a charged-particle. A Lagrangian formalism based on the action principle has been suggested [1]- [3]. A Hamiltonian formalism is considered in [4]– [5]. In the case of slowly varying electromagnetic fields an approach based on the Dirac equation *via* the WKB approximation scheme has been presented [6]. In the context of the Aharonov-Bohm and Aharonov-Casher effects [7]- [8], *the question as to whether neutron acceleration can occur in uniform electromagnetic fields is also raised* [9]- [11]. In the very recent work of Chaichian [4] it has been rightly pointed out that in the nonrelativistic limit the results of the above approaches do not coincide. This motivates us to examine the form of the force derived from the Dirac equation using the FW-transformation [13]- [14] scheme; note that the FW-transformation technique is the only one in which we can take the meaningful nonrelativistic limit of the Dirac equation [15]. The FW-approach gives the expression for the force in the presence of external time-dependent fields, the nonrelativistic limit and a systematic procedure to obtain the relativistic corrections to a desired degree of accuracy. In such a derivation we also take into account the anomalous electric moment. We compare the results of our derivation with other approaches mentioned above. One should also note that a novel approach for producing polarized beams has been suggested using the Stern-Gerlach forces [16]- [17].

II. SECTION

Let us consider the Dirac particle of rest mass m_0, charge q, anomalous magnetic moment μ_a and anomalous electric moment ϵ_a. In presence of the external electromagnetic fields, the Dirac equation is

$$i\hbar\frac{\partial}{\partial t}\left|\Psi_D\right\rangle = \hat{\mathrm{H}}_D\left|\Psi_D\right\rangle \tag{1}$$

and the Dirac Hamiltonian H_D, including the contributions of the anomalous magnetic moment and anomalous electric moment is given by [18]:

$$\hat{\mathrm{H}}_D = \beta m_0 c^2 + \hat{\mathcal{E}} + \hat{\mathcal{O}} \tag{2}$$

$$\hat{\mathcal{E}} = +q\phi(\boldsymbol{r})I - \mu_a\beta\boldsymbol{\Sigma}\cdot\boldsymbol{B} + \epsilon_a\beta\boldsymbol{\Sigma}\cdot\boldsymbol{E} \tag{3}$$

$$\hat{\mathcal{O}} = c\boldsymbol{\alpha}\cdot(-i\hbar\boldsymbol{\nabla} - q\boldsymbol{A}) + i\mu_a\beta\boldsymbol{\alpha}\cdot\boldsymbol{E} + i\epsilon_a\beta\boldsymbol{\alpha}\cdot\boldsymbol{B}$$

$$\beta = \begin{pmatrix} \mathbb{1} & \mathbf{0} \\ \mathbf{0} & -\mathbb{1} \end{pmatrix}, \quad \boldsymbol{\alpha} = \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & \mathbf{0} \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma} \end{pmatrix}, \tag{4}$$

where $\boldsymbol{\sigma}$ is the triplet of Pauli matrices.

In the nonrelativistic situation the upper pair of components of the Dirac spinor Ψ_D are large compared to the lower pair of components. The operator $\hat{\mathcal{E}}$ which does not couple the large and small components of Ψ_D is called as *even* and $\hat{\mathcal{O}}$ is called as an *odd* operator which couples the large to small components. Note that $\beta\hat{\mathcal{O}} = -\hat{\mathcal{O}}\beta$ and $\beta\hat{\mathcal{E}} = \hat{\mathcal{E}}\beta$. This motivates us to look for a transformation which will eliminate the odd-part completely from

the Dirac Hamiltonian. Such a transformation is available in the case of the free-particle. In the very general case of time-dependent fields such a transformation is not known to exist. Therefore, one has to be content with an approximation procedure which reduces the strength of the odd-part to a desired degree of accuracy in powers of $\frac{1}{m_0 c^2}$. We shall follow the Foldy-Wouthuysen [13]- [14] transformation technique to take the nonrelativistic limit of the Dirac Hamiltonian in (4) to reduce the strength of the odd-part in power series in $\frac{1}{m_0 c^2}$. The result to the leading order, that is to order $\frac{1}{m_0 c^2}$ is formally given by

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{\mathcal{H}}^{(2)} |\psi\rangle \,,$$
$$\hat{\mathcal{H}}^{(2)} = m_0 c^2 \beta + \hat{\mathcal{E}} + \frac{1}{2m_0 c^2} \beta \hat{\mathcal{O}}^2 \tag{5}$$

and to next higher, order $\frac{1}{(m_0 c^2)^3}$ is given by

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{\mathcal{H}}^{(4)} |\psi\rangle \,,$$
$$\hat{\mathcal{H}}^{(4)} = m_0 c^2 \beta + \hat{\mathcal{E}} + \frac{1}{2m_0 c^2} \beta \hat{\mathcal{O}}^2$$
$$- \frac{1}{8m_0^2 c^4} \left[\hat{\mathcal{O}}, \left(\left[\hat{\mathcal{O}}, \hat{\mathcal{E}} \right] + i\hbar \frac{\partial}{\partial t} \hat{\mathcal{O}} \right) \right] - \frac{1}{(2m_0 c^2)^3} \beta \hat{\mathcal{O}}^4 \,. \tag{6}$$

A detailed discussion of the FW-transformation and the derivation of the above Hamiltonians can be found in many places (for instance the book by Bjorken and Drell in [14]).

As a first step we consider the case of a charged particle neglecting the anomalous moments. In this case the Hamiltonian (6) works out to

$$\hat{\mathcal{H}}^{(4)} = m_0 c^2 + q\phi + \frac{1}{2m_0} \left(\hat{\pi}^2 - q\hbar \boldsymbol{\sigma} \cdot \boldsymbol{B} \right)$$
$$+ \frac{1}{8m_0^2 c^2} \hbar q \boldsymbol{\sigma} \cdot (\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}})$$
$$- \frac{1}{8m_0^3 c^2} \left\{ \hat{\pi}^4 + \hbar^2 q^2 B^2 - \hbar q \left(\hat{\pi}^2 \left(\boldsymbol{\sigma} \cdot \boldsymbol{B} \right) + \left(\boldsymbol{\sigma} \cdot \boldsymbol{B} \right) \hat{\pi}^2 \right) \right\} \tag{7}$$

A detailed formula including the μ_a contibutions is given by (A1) in the appendix.

III. SECTION

Now we use the Hamiltonian derived in (7) to compute the acceleration, $\boldsymbol{a}$ experienced by the particle using the Heisenberg representation,

$$\frac{d}{dt} \langle \hat{\mathcal{O}} \rangle = \frac{i}{\hbar} \langle \left[\hat{\mathcal{H}}, \hat{\mathcal{O}} \right] \rangle + \left\langle \frac{\partial}{\partial t} \hat{\mathcal{O}} \right\rangle \,. \tag{8}$$

and we omit the brackets $\langle \cdots \rangle$. Then we obtain,

$$m_0\dot{\boldsymbol{r}} = m_0\frac{d}{dt}\boldsymbol{r} = m_0\frac{\mathrm{i}}{\hbar}\left[\hat{\mathcal{H}},\boldsymbol{r}\right]$$

$$= \hat{\boldsymbol{\pi}} - \frac{1}{4m_0^2c^2}\left(\hat{\pi}^2\hat{\boldsymbol{\pi}} + \hat{\boldsymbol{\pi}}\hat{\pi}^2\right) - \frac{\hbar q}{4m_0c^2}\left(\boldsymbol{\sigma}\times\boldsymbol{E}\right) + \frac{\hbar q}{4m_0^2c^2}\left(\hat{\boldsymbol{\pi}}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) + \left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\hat{\boldsymbol{\pi}}\right) \quad (9)$$

Using the above expression for $\dot{\boldsymbol{r}}$ we compute the acceleration

$$m_0\boldsymbol{a} = m_0\frac{d}{dt}\dot{\boldsymbol{r}} = m_0\ddot{\boldsymbol{r}}$$

$$= q\boldsymbol{E} - \frac{q}{4m_0^2c^2}\left(\hat{\pi}^2\boldsymbol{E} + \boldsymbol{E}\hat{\pi}^2\right)$$

$$- \frac{q}{2m_0}\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{B} - \boldsymbol{B}\times\hat{\boldsymbol{\pi}}\right)$$

$$- \frac{q}{4m_0^3c^2}\left(\hat{\pi}^2\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{B} - \boldsymbol{B}\times\hat{\boldsymbol{\pi}}\right) + \left(\hat{\boldsymbol{\pi}}\times\boldsymbol{B} - \boldsymbol{B}\times\hat{\boldsymbol{\pi}}\right)\hat{\pi}^2\right)$$

$$- \frac{q}{4m_0^2c^2}\left\{\left(\hat{\boldsymbol{\pi}}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\hat{\boldsymbol{\pi}}\right)\hat{\boldsymbol{\pi}} + \hat{\boldsymbol{\pi}}\left(\hat{\boldsymbol{\pi}}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\hat{\boldsymbol{\pi}}\right)\right\}$$

$$+ \frac{\hbar q}{2m_0}\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) - \frac{\hbar q}{8m_0^3c^2}\left(\hat{\pi}^2\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) + \boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\hat{\pi}^2\right)$$

$$+ \frac{\hbar q^2}{4m_0^2c^2}\left(\boldsymbol{E}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) + \left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\boldsymbol{E}\right) - \frac{\hbar q}{8m_0^2c^2}\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}\right)\right)$$

$$+ \frac{\hbar q^2}{8m_0^3c^2}\left\{\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{B} - \boldsymbol{B}\times\hat{\boldsymbol{\pi}}\right)\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) + \left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{B} - \boldsymbol{B}\times\hat{\boldsymbol{\pi}}\right)\right\}$$

$$+ \frac{\hbar^2 q^2}{8m_0^3c^2}\boldsymbol{\nabla}\left(B^2\right) + \frac{\hbar q^2}{4m_0^2c^2}\boldsymbol{E}\left(\boldsymbol{\sigma}\times\boldsymbol{B}\right)$$

$$- \frac{\hbar q}{4m_0c^2}\frac{\partial}{\partial t}\left(\boldsymbol{\sigma}\times\boldsymbol{B}\right) + \frac{\hbar q}{4m_0^2c^2}\left(\hat{\boldsymbol{\pi}}\frac{\partial}{\partial t}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) + \frac{\partial}{\partial t}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)\hat{\boldsymbol{\pi}}\right)$$

$$+ \boldsymbol{R} \quad (10)$$

where the r_k-th component of $\boldsymbol{R}$ is

$$\left(\boldsymbol{R}\right)_{r_k} = -\frac{\hbar q}{8m_0^2c^2}\left\{\hat{\boldsymbol{\pi}}\cdot\boldsymbol{\nabla}\left(\left(\boldsymbol{\sigma}\times\boldsymbol{E}\right)_{r_k}\right) + \boldsymbol{\nabla}\left(\left(\boldsymbol{\sigma}\times\boldsymbol{E}\right)_{r_k}\right)\cdot\hat{\boldsymbol{\pi}}\right\}$$

$$r_k = x\,,y\,,z\,,\quad k = 1\,,2\,,3\,. \quad (11)$$

The above expression for the acceleration can be related to the classical expression when we make the substitution $\frac{\hat{\boldsymbol{\pi}}}{m_0} \longrightarrow \boldsymbol{v}$ where $\boldsymbol{v}$ is the velocity of the particle. With such a substitution and with $\beta = \frac{|\boldsymbol{v}|}{c}$ we get,

$$\boldsymbol{a} = \left(1 - \frac{1}{2}\beta^2\right)\frac{q}{m_0}\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}\right) - \frac{q}{m_0}\frac{\boldsymbol{v}}{c^2}\left(\boldsymbol{v}\cdot\boldsymbol{E}\right)$$

$$\left(1 - \frac{1}{2}\beta^2\right)\frac{\hbar q}{2m_0}\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) - \frac{\hbar q}{4m_0c^2}\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\left(\boldsymbol{v}\times\boldsymbol{E}\right)\right)$$

$$\frac{1}{m_0c^2}\left\{q\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}\right)\right\}\frac{\hbar q}{2m_0}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right)$$

$$+ \cdots \quad (12)$$

The above derivation is consistent with the result [19] of classical electrodynamics

$$\boldsymbol{a} = \frac{q}{m_0}\sqrt{1-\beta^2}\left\{\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B} - \frac{\boldsymbol{v}}{c^2}\left(\boldsymbol{v}\cdot\boldsymbol{E}\right)\right\} \tag{13}$$

In the nonrelativistic limit the force $\boldsymbol{F}$ is well-approximated by the expression $\boldsymbol{F} = m_0\boldsymbol{a}$. So we can use the expression for the acceleration, $\boldsymbol{a}$ derived using the Foldy-Wouthuysen technique to express the force experienced by the charged particle.

The leading order Foldy-Wouthuysen Hamiltonian, when we take the anomalous magnetic moment and anomalous electric moment into account is given by

$$\hat{\mathcal{H}}^{(2)} = m_0c^2 - \mu_a\boldsymbol{\sigma}\cdot\boldsymbol{B} + \epsilon_a\boldsymbol{\sigma}\cdot\boldsymbol{E} + q\phi$$
$$+ \frac{1}{2m_0c^2}\left\{c^2\left(\hat{\pi}^2 - q\hbar\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) + \left(\mu_a\boldsymbol{E} + \epsilon_a\boldsymbol{B}\right)^2\right.$$
$$\left. + \mu_a c\boldsymbol{\sigma}\cdot\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}\right) + \epsilon_a c\boldsymbol{\sigma}\cdot\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{B} - \boldsymbol{B}\times\hat{\boldsymbol{\pi}}\right)\right\} \tag{14}$$

Next, to leading order Hamiltonian is given in (A1) of the appendix.

Now we use the above derived Hamiltonians in (14) to compute the Lorentz-Stern-Gerlach force and we get

$$\dot{\boldsymbol{r}} = \frac{d}{dt}\boldsymbol{r} = \frac{1}{m_0}\left\{\hat{\boldsymbol{\pi}} - \left(\frac{\mu_a}{c}\left(\boldsymbol{\sigma}\times\boldsymbol{E}\right) + \frac{\epsilon_a}{c}\left(\boldsymbol{\sigma}\times\boldsymbol{B}\right)\right)\right\}$$
$$= \frac{1}{m_0}\hat{\Pi} \tag{15}$$

Where $\hat{\Pi}$ is the *kinetic momentum*. The Lorentz-Stern-Gerlach force in the absence of ϵ_a is:

$$\boldsymbol{F} = \frac{d}{dt}\hat{\Pi} = \frac{\mathrm{i}}{\hbar}\left[\hat{\mathcal{H}},\hat{\Pi}\right] + \frac{\partial}{\partial t}\hat{\Pi}$$
$$= q\left\{\boldsymbol{E} + \frac{1}{2m_0}\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{B} - \boldsymbol{B}\times\hat{\boldsymbol{\pi}}\right)\right\}$$
$$+ \left(\mu_a + \frac{q\hbar}{2m_0}\right)\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\boldsymbol{B}\right) - \frac{\mu_a}{c}\frac{\partial}{\partial t}\left(\boldsymbol{\sigma}\times\boldsymbol{E}\right)$$
$$- \frac{\mu_a}{2m_0c}\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}\right)\right)$$
$$+ 2\frac{\mu_a}{\hbar c}\left(\mu_a + \frac{\hbar q}{2m_0}\right)\boldsymbol{E}\times\left(\boldsymbol{\sigma}\times\boldsymbol{B}\right) - \frac{1}{2m_0c^2}\boldsymbol{\nabla}\left(\mu_a^2\boldsymbol{E}^2\right)$$
$$+ \mathrm{i}\frac{\mu_a^2}{2m_0c^2\hbar}\left(\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{E}\right)\times\boldsymbol{E} - \boldsymbol{E}\times\left(\boldsymbol{E}\times\hat{\boldsymbol{\pi}}\right)\right)$$
$$+ \frac{\mu_a^2}{2m_0c^2\hbar}\left\{\left(\boldsymbol{\sigma}\times\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}\right)\right)\times\boldsymbol{E} - \boldsymbol{E}\times\left(\boldsymbol{\sigma}\times\left(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}\right)\right)\right\}$$
$$+ \boldsymbol{R} \tag{16}$$

where the r_k-th component of $\boldsymbol{R}$ is

$$(\boldsymbol{R})_{r_k} = -\frac{\mu_a}{2m_0 c} \left\{ \hat{\boldsymbol{\pi}} \cdot \boldsymbol{\nabla} \left((\boldsymbol{\sigma} \times \boldsymbol{E})_{r_k} \right) + \boldsymbol{\nabla} \left((\boldsymbol{\sigma} \times \boldsymbol{E})_{r_k} \right) \cdot \hat{\boldsymbol{\pi}} \right\}$$
$$r_k = x, y, z, \quad k = 1, 2, 3. \tag{17}$$

For simplicity we first consider the the acceleration (or equivalently the force) experienced by a neutron

$$\begin{aligned}
\boldsymbol{F}_{neutron} = &+\mu_a \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot \boldsymbol{B} \right) - \frac{\mu_a}{c} \frac{\partial}{\partial t} \left(\boldsymbol{\sigma} \times \boldsymbol{E} \right) \\
&- \frac{\mu_a}{2m_0 c} \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}}) \right) \\
&+ 2\frac{\mu_a^2}{\hbar c} \boldsymbol{E} \times (\boldsymbol{\sigma} \times \boldsymbol{B}) - \frac{1}{2m_0 c^2} \boldsymbol{\nabla} \left(\mu_a^2 \boldsymbol{E}^2 \right) \\
&+ i\frac{\mu_a^2}{2m_0 c^2 \hbar} \left((\hat{\boldsymbol{p}} \times \boldsymbol{E}) \times \boldsymbol{E} - \boldsymbol{E} \times (\boldsymbol{E} \times \hat{\boldsymbol{p}}) \right) \\
&+ \frac{\mu_a^2}{2m_0 c^2 \hbar} \left\{ (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}})) \times \boldsymbol{E} - \boldsymbol{E} \times (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}})) \right\} \\
&+ \cdots \\
&+ \mathrm{O}\left(\mu_a^3 \right)
\end{aligned} \tag{18}$$

In the above expression in (18) the leading terms have been retained and the "$\cdots$" indicates the higher order terms. The complete expression is given in (A2) in the appendix. The detailed formulae shall be given in an appendix at the end. This is the case where ever the "$\cdots$" appear in the expressions.

From the expression (18) we conclude that the leading order (linear in μ_a) contributions to the neutron acceleration come through the gradients and the time derivatives of the electromagnetic fields. Such contributions disappear in the case of uniform and constant fields respectively. The next-to-leading order contributions come from the terms of the type $\mu_a^2 \boldsymbol{E} \times (\boldsymbol{\sigma} \times \boldsymbol{B})$. Such contributions do not vanish and hence we have neutron acceleration even in the presense of uniform fields. Such accelerations are quadratic (and higher powers) in μ_a and hence are very small.

In the expression (16) for the Lorentz-Stern-Gerlach force if we substitute $\mu_a = g\frac{\hbar|q|}{4m_0}$ and $q = -e$ we get the often mentioned term, $+g(g-2)\boldsymbol{E} \times (\boldsymbol{\sigma} \times \boldsymbol{B})$ in [6].

In the presence of the anomalous electric moment ϵ_a the Lorentz-Stern-Gerlach force is:

$$\begin{aligned}
\boldsymbol{F} = &\, q \left\{ \boldsymbol{E} + \frac{1}{2m_0} \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}} \right) \right\} \\
&+ \left(\mu_a + \frac{q\hbar}{2m} \right) \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot \boldsymbol{B} \right) - \epsilon_a \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot \boldsymbol{E} \right) \\
&- \frac{1}{c} \frac{\partial}{\partial t} \left(\mu_a (\boldsymbol{\sigma} \times \boldsymbol{E}) + \epsilon_a (\boldsymbol{\sigma} \times \boldsymbol{B}) \right) \\
&- \frac{\mu_a}{2m_0 c} \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot (\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}) \right) - \frac{\epsilon_a}{2m_0 c} \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot (\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}) \right) \\
&\cdots
\end{aligned} \tag{19}$$

IV. CONCLUSIONS AND SUMMARY

As can be seen above we get a variety of terms contributing to the Lorentz-Stern-Gerlach force.

The nonrelativistic static limit coincides with the usual "classical" formula if $\boldsymbol{B}$ is time-independent, inhomogeneous and $\boldsymbol{E}$ is absent in the lab system. Otherwise there are differences even at low non-relativistic velocities. In particular one may consider the force terms $\frac{\mu_a}{2m_0 c}\boldsymbol{\nabla}\left(\boldsymbol{\sigma}\cdot(\boldsymbol{v}\times\boldsymbol{E})\right)$ and $\frac{1}{2m_0 c^2}\boldsymbol{\nabla}\left(\mu_a^2\boldsymbol{E}^2\right)$ that are present whenever a spin-$\frac{1}{2}$ particle with charge enters into an inhomogeneous static electric field (in absence of $\boldsymbol{B}$).

Another relevant point to be noted is the force experienced by a neutron (more generally by an electrically neutral particle). In this case we find contributions even when the fields are homogeneous and static.

The LSG force derived using the FW-tchnique differs from the other approaches which use a "classical" or "semiclassical" treatment of the relativistic Stern-Gerlach force [5].

Only experiments with very high precision can conclude about the finer differences in the various expressions for the force.

Acknowledgement

APPENDIX

For the general case including, the contributions of the anomalous magnetic moment, the Hamiltonian (in 6) works out to

$$
\begin{aligned}
\hat{\mathcal{H}}^{(4)} = {}& m_0 c^2 + q\phi + \frac{1}{2m_0}\hat{\pi}^2 \\
& - \left(\frac{\hbar q}{2m_0} + \mu_a\right)(\boldsymbol{\sigma}\cdot\boldsymbol{B}) + \frac{\mu_a}{2m_0 c}\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}) + \frac{\mu_a^2}{2m_0 c^2}E^2 \\
& + \frac{1}{8m_0^2 c^4}\Big\{ +\hbar q c^2 \boldsymbol{\sigma}\cdot(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}) + 2\mu_a \hbar q c E^2 \\
& \qquad\qquad -\mu_a^2\left((\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\pi}})(\hat{\boldsymbol{\pi}}\cdot\boldsymbol{B} + \boldsymbol{B}\cdot\hat{\boldsymbol{\pi}}) + (\hat{\boldsymbol{\pi}}\cdot\boldsymbol{B} + \boldsymbol{B}\cdot\hat{\boldsymbol{\pi}})(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\pi}})\right) \\
& \qquad\qquad -\mu_a^2\hbar c\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}(\boldsymbol{E}\cdot\boldsymbol{B} + \boldsymbol{B}\cdot\boldsymbol{E})) + 4\mu_a^3(\boldsymbol{\sigma}\cdot\boldsymbol{E})(\boldsymbol{E}\cdot\boldsymbol{B})\Big\} \\
& - \frac{1}{(2m_0 c^2)^3}\Big\{ c^4\left(\hat{\pi}^4 + \hbar^2 q^2 B^2 - \hbar q\left(\hat{\pi}^2(\boldsymbol{\sigma}\cdot\boldsymbol{B}) + (\boldsymbol{\sigma}\cdot\boldsymbol{B})\hat{\pi}^2\right)\right) \\
& \qquad\qquad + \mu_a c^3\left(\hat{\pi}^2\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}) + \boldsymbol{\sigma}\cdot(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}})\hat{\pi}^2\right) \\
& \qquad\qquad - \mu_a c\hbar q\left(\boldsymbol{B}\cdot(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}) + (\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}})\cdot\boldsymbol{B}\right) \\
& \qquad\qquad - \mathrm{i}\mu_a c\hbar q\boldsymbol{\sigma}\cdot\left(\boldsymbol{B}\times(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}}) + (\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}})\times\boldsymbol{B}\right) \\
& \qquad\qquad + \mu_a^2 c^2\left(\left(\hat{\pi}^2 E^2 + E^2\hat{\pi}^2\right) + (\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}})^2\right) \\
& \qquad\qquad + \mathrm{i}\mu_a^2 c^2\boldsymbol{\sigma}\cdot\left((\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}})\times(\hat{\boldsymbol{\pi}}\times\boldsymbol{E} - \boldsymbol{E}\times\hat{\boldsymbol{\pi}})\right) \\
& \qquad\qquad - \mu_a^2 c^2\hbar q\left(E^2(\boldsymbol{\sigma}\cdot\boldsymbol{B}) + (\boldsymbol{\sigma}\cdot\boldsymbol{B})E^2\right)
\end{aligned}
$$

$$+\mu_a^3 c \left(E^2 \boldsymbol{\sigma} \cdot (\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}) + \boldsymbol{\sigma} \cdot (\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}) E^2 \right)$$

$$+\mu_a^4 E^4 \Big\} \, . \tag{A1}$$

The total acceleration (or equivalently the force) experienced by a neutron when both μ_a and ϵ_a are taken into account is:

$$
\begin{aligned}
\boldsymbol{F} = {} & \mu_a \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot \boldsymbol{B} \right) - \epsilon_a \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot \boldsymbol{E} \right) \\
& - \frac{1}{c} \frac{\partial}{\partial t} \left(\mu_a \left(\boldsymbol{\sigma} \times \boldsymbol{E} \right) + \epsilon_a \left(\boldsymbol{\sigma} \times \boldsymbol{B} \right) \right) \\
& - \frac{\mu_a}{2 m_0 c} \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}}) \right) - \frac{\epsilon_a}{2 m_0 c} \boldsymbol{\nabla} \left(\boldsymbol{\sigma} \cdot (\hat{\boldsymbol{p}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{p}}) \right) \\
& - \frac{1}{2 m_0 c^2} \boldsymbol{\nabla} \left\{ \mu_a^2 \boldsymbol{E}^2 + \epsilon_a^2 \boldsymbol{B}^2 + \mu_a \epsilon_a \left(\boldsymbol{E} \cdot \boldsymbol{B} + \boldsymbol{B} \cdot \boldsymbol{E} \right) \right\} \\
& - 2 \frac{\mu_a^2}{\hbar c} \left(\boldsymbol{\sigma} \times \boldsymbol{B} \right) \times \boldsymbol{E} - 2 \frac{\epsilon_a^2}{\hbar c} \left(\boldsymbol{\sigma} \times \boldsymbol{E} \right) \times \boldsymbol{B} \\
& + 2 \frac{\mu_a \epsilon_a}{\hbar c} \left((\boldsymbol{\sigma} \times \boldsymbol{E}) \times \boldsymbol{E} - (\boldsymbol{\sigma} \times \boldsymbol{B}) \times \boldsymbol{B} \right) \\
& - \mathrm{i} \frac{\mu_a^2}{2 m_0 c^2 \hbar} \left((\hat{\boldsymbol{p}} \times \boldsymbol{E}) \times \boldsymbol{E} - \boldsymbol{E} \times (\boldsymbol{E} \times \hat{\boldsymbol{p}}) \right) \\
& - \mathrm{i} \frac{\epsilon_a^2}{2 m_0 c^2 \hbar} \left((\hat{\boldsymbol{p}} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{B} \times (\boldsymbol{B} \times \hat{\boldsymbol{p}}) \right) \\
& + \frac{\mu_a^2}{2 m_0 c^2 \hbar} \left\{ (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}})) \times \boldsymbol{E} - \boldsymbol{E} \times (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}})) \right\} \\
& + \frac{\epsilon_a^2}{2 m_0 c^2 \hbar} \left\{ (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{p}})) \times \boldsymbol{B} - \boldsymbol{B} \times (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{p}})) \right\} \\
& - \mathrm{i} \frac{\mu_a \epsilon_a}{2 m_0 c^2 \hbar} \left\{ (\hat{\boldsymbol{p}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{p}}) \times \boldsymbol{E} + \boldsymbol{E} \times (\hat{\boldsymbol{p}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{p}}) \right. \\
& \qquad\qquad\qquad \left. + (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}}) \times \boldsymbol{B} + \boldsymbol{B} \times (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}}) \right\} \\
& + \frac{\mu_a \epsilon_a}{2 m_0 c^2 \hbar} \left\{ (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{p}})) \times \boldsymbol{E} - \boldsymbol{E} \times (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{p}})) \right. \\
& \qquad\qquad\qquad \left. + (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}})) \times \boldsymbol{B} - \boldsymbol{B} \times (\boldsymbol{\sigma} \times (\hat{\boldsymbol{p}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{p}})) \right\} \\
& + \boldsymbol{R}
\end{aligned}
$$

$$\tag{A2}$$

where the r_k-th component of $\boldsymbol{R}$ is

$$
\begin{aligned}
(\boldsymbol{R})_{r_k} = {} & - \frac{\mu_a}{2 m_0 c} \left\{ \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla} \left((\boldsymbol{\sigma} \times \boldsymbol{E})_{r_k} \right) + \boldsymbol{\nabla} \left((\boldsymbol{\sigma} \times \boldsymbol{E})_{r_k} \right) \cdot \hat{\boldsymbol{p}} \right\} \\
& - \frac{\epsilon_a}{2 m_0 c} \left\{ \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla} \left((\boldsymbol{\sigma} \times \boldsymbol{B})_{r_k} \right) + \boldsymbol{\nabla} \left((\boldsymbol{\sigma} \times \boldsymbol{B})_{r_k} \right) \cdot \hat{\boldsymbol{p}} \right\} \\
& r_k = x \, , y \, , z \, , \quad k = 1 \, , 2 \, , 3 \, . \tag{A3}
\end{aligned}
$$

In the presence of the anomalous electric moment ϵ_a the Lorentz-Stern-Gerlach force is:

$$\boldsymbol{F} = q\left\{\boldsymbol{E} + \frac{1}{2m_0}\left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right\}$$

$$+ \left(\mu_a + \frac{q\hbar}{2m}\right)\boldsymbol{\nabla}\left(\boldsymbol{\sigma} \cdot \boldsymbol{B}\right) - \epsilon_a \boldsymbol{\nabla}\left(\boldsymbol{\sigma} \cdot \boldsymbol{E}\right)$$

$$- \frac{1}{c}\frac{\partial}{\partial t}\left(\mu_a\left(\boldsymbol{\sigma} \times \boldsymbol{E}\right) + \epsilon_a\left(\boldsymbol{\sigma} \times \boldsymbol{B}\right)\right)$$

$$- \frac{\mu_a}{2m_0 c}\boldsymbol{\nabla}\left(\boldsymbol{\sigma} \cdot \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right)\right) - \frac{\epsilon_a}{2m_0 c}\boldsymbol{\nabla}\left(\boldsymbol{\sigma} \cdot \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right)$$

$$- \frac{1}{2m_0 c^2}\boldsymbol{\nabla}\left\{\mu_a^2 \boldsymbol{E}^2 + \epsilon_a^2 \boldsymbol{B}^2 + \mu_a\epsilon_a\left(\boldsymbol{E} \cdot \boldsymbol{B} + \boldsymbol{B} \cdot \boldsymbol{E}\right)\right\}$$

$$+ \frac{\mu_a q}{m_0 c}\left(\left(\boldsymbol{\sigma} \times \boldsymbol{E}\right) \times \boldsymbol{B} - \left(\boldsymbol{\sigma} \times \boldsymbol{B}\right) \times \boldsymbol{E}\right)$$

$$- 2\frac{\mu_a^2}{\hbar c}\left(\boldsymbol{\sigma} \times \boldsymbol{B}\right) \times \boldsymbol{E} - 2\frac{\epsilon_a^2}{\hbar c}\left(\boldsymbol{\sigma} \times \boldsymbol{E}\right) \times \boldsymbol{B}$$

$$+ 2\frac{\mu_a\epsilon_a}{\hbar c}\left(\left(\boldsymbol{\sigma} \times \boldsymbol{E}\right) \times \boldsymbol{E} - \left(\boldsymbol{\sigma} \times \boldsymbol{B}\right) \times \boldsymbol{B}\right)$$

$$- \mathrm{i}\frac{\mu_a^2}{2m_0 c^2 \hbar}\left(\left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E}\right) \times \boldsymbol{E} - \boldsymbol{E} \times \left(\boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right)\right)$$

$$- \mathrm{i}\frac{\epsilon_a^2}{2m_0 c^2 \hbar}\left(\left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B}\right) \times \boldsymbol{B} - \boldsymbol{B} \times \left(\boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right)$$

$$+ \frac{\mu_a^2}{2m_0 c^2 \hbar}\left\{\left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right)\right) \times \boldsymbol{E} - \boldsymbol{E} \times \left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right)\right)\right\}$$

$$+ \frac{\epsilon_a^2}{2m_0 c^2 \hbar}\left\{\left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right) \times \boldsymbol{B} - \boldsymbol{B} \times \left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right)\right\}$$

$$- \mathrm{i}\frac{\mu_a\epsilon_a}{2m_0 c^2 \hbar}\left\{\left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right) \times \boldsymbol{E} + \boldsymbol{E} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right.$$

$$\left. + \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right) \times \boldsymbol{B} + \boldsymbol{B} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right)\right\}$$

$$+ \frac{\mu_a\epsilon_a}{2m_0 c^2 \hbar}\left\{\left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right) \times \boldsymbol{E} - \boldsymbol{E} \times \left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{B} - \boldsymbol{B} \times \hat{\boldsymbol{\pi}}\right)\right)\right.$$

$$\left. + \left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right)\right) \times \boldsymbol{B} - \boldsymbol{B} \times \left(\boldsymbol{\sigma} \times \left(\hat{\boldsymbol{\pi}} \times \boldsymbol{E} - \boldsymbol{E} \times \hat{\boldsymbol{\pi}}\right)\right)\right\}$$

$$+ \boldsymbol{R} \tag{A4}$$

where the r_k-th component of $\boldsymbol{R}$ is

$$\left(\boldsymbol{R}\right)_{r_k} = -\frac{\mu_a}{2m_0 c}\left\{\hat{\boldsymbol{\pi}} \cdot \boldsymbol{\nabla}\left(\left(\boldsymbol{\sigma} \times \boldsymbol{E}\right)_{r_k}\right) + \boldsymbol{\nabla}\left(\left(\boldsymbol{\sigma} \times \boldsymbol{E}\right)_{r_k}\right) \cdot \hat{\boldsymbol{\pi}}\right\}$$

$$- \frac{\epsilon_a}{2m_0 c}\left\{\hat{\boldsymbol{\pi}} \cdot \boldsymbol{\nabla}\left(\left(\boldsymbol{\sigma} \times \boldsymbol{B}\right)_{r_k}\right) + \boldsymbol{\nabla}\left(\left(\boldsymbol{\sigma} \times \boldsymbol{B}\right)_{r_k}\right) \cdot \hat{\boldsymbol{\pi}}\right\}$$

$$r_k = x, y, z, \quad k = 1, 2, 3. \tag{A5}$$

As can be seen above we get a variety of terms contributing to the Lorentz-Stern-Gerlach force.

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